BRL

MEMORANDUM REPORT NO. 1352
JUNE 1961

DYNAMIC STABILITY MEASUREMENTS

T. L. Smith

Department of the Army Project No. 503-03-009
Ordnance Management Structure Code No. 5210.11.140

BALLISTIC RESEARCH LABORATORIES

ABERDEEN PROVING GROUND, MARYLAND

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A B E R D E E N   P R O V I N G   G R O U N D,   M A R Y L A N D

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SYMBOLS

f	frequency of oscillations, cycles per second
I	moment of inertia, slugs ft^2
k	rotational spring constant ft. lb/radian
k_1	mechanical spring constant of model support system
k_o	aerodynamic moment coefficient such that $-k_o\theta$ is that part of the aerodynamic moment proportional to θ
k_2	$k_1 + k_o$
P	period of oscillation, sec, equal to $1/f$
t	time in seconds
λ	reciprocal of the time constant for exponential decay, sec^{-1}
μ	viscous damping coefficient, ft. lb. per radian per second
μ_o	aerodynamic damping coefficient
μ_1	equivalent mechanical viscous damping coefficient for tare damping
μ_2	$\mu_1 + \mu_o$
θ	angle of attack of oscillating model
$\theta_o, \theta_1, \theta_2, \ldots, \theta_n \ldots$	successive maxima of θ, occurring at times $t_o, t_1, t_2, \ldots, t_n, \ldots$
ω	angular velocity of phase angle of the oscillation
ρ	air density in test section, slugs per cubic foot
V	air velocity in test section, feet per second
q	dynamic pressure $= \frac{1}{2}\rho V^2$, lbs. per sq. ft.
d	model reference diameter, feet
S	model reference area $= \frac{\pi d^2}{4}$, ft^2

DIFFERING SYMBOL CONVENTIONS

The aerodynamic parts μ_o and k_o of the damping and spring parameters (due to air forces), or damping in pitch and static stability parameters, are dimensional parameters related to corresponding non-dimensional coefficients. Unfortunately, there are several definitions of these coefficients used in range and missile literature and reports, for rotationally symmetric missiles, including:

A. An aerodynamic form closely resembling the coefficients used by the aircraft industry, using C_{M_α} and $C_{M_q} + C_{M_{\dot{\alpha}}}$ for the static stability and for the damping in pitch coefficients. This form is used by the Naval Ordnance Laboratory[5], many missile contractors for the three services, and in earlier reports by the Ballistic Research Laboratories.

B. Exactly the same symbols are used, for coefficients differing in sign for normal force coefficients and differing by a factor of 1/2 for coefficients related to angular velocities, from the coefficients referred to in A above. This type of coefficient is now used by NASA, by Charles Murphy of BRL since 1957[6], and was first used by a data report of the Free Flight Aerodynamics Branch of BRL[7] in 1960.

C. The ballistic coefficients used in the theory of flight of shells for many years by Fowler; Kent; Kelly, McShane and Reno, and others.

To define these coefficients, let positive moment on the model be a moment in the positive direction of measurement of the angle α of pitch. The moment on the model due to aerodynamic forces is then

$$\text{moment} = -\mu_o \dot{\alpha} - k_o \alpha$$

so that positive μ_o means a moment opposing the angular velocity and hence positive damping or absorption of energy, and positive k_o means static stability, a moment toward the position $\alpha = 0$.

In terms of the A type coefficients, the moment is

$$\text{moment} = \frac{1}{2}\rho V^2 (Sd) \frac{\dot{\alpha} d}{2V} (C_{M_q}^{A} + C_{M_{\dot{\alpha}}}^{A}) + \frac{1}{2}\rho V^2 (Sd) \alpha C_{M_\alpha}^{A}$$

while in terms of the B type coefficient,

$$\text{moment} = \frac{1}{2}\rho V^2 (Sd) (\frac{\dot{\alpha} d}{V}) (C_{M_q}^{B} + C_{M_{\dot{\alpha}}}^{B}) + (\frac{1}{2}\rho V^2) Sd) \alpha C_{M_\alpha}^{B} \quad .$$

In terms of the ballistic coefficients,

$$\text{moment} = -\rho V d^4 \dot{\alpha} K_H + \rho V^2 d^3 \alpha K_M \quad .$$

Relations between the coefficients are

$$\frac{1}{2}(C_{M_q}^{A} + C_{M_{\dot{\alpha}}}^{A}) = (C_{M_q}^{B} + C_{M_{\dot{\alpha}}}^{B}) = -\frac{8}{\pi} K_H = -\frac{8}{\pi \rho V d^4} \mu_o$$

$$C_{M_\alpha}^{A} = C_{M_\alpha}^{B} = \frac{8}{\pi} K_M = -\frac{8}{\pi \rho V^2 d^3} k_o \quad .$$

SUMMARY

This paper is a short discussion of the determination of dynamic and static stability of a non-spinning missile in a wind tunnel. It is assumed that the method used is to mount the model so that it is free to oscillate, and to observe the time rate of decay of oscillations with and without the wind on. Some of the sources of error are discussed.

TYPE OF TEST

It is assumed that the test is to study the dynamic stability of a non-spinning missile when flying at supersonic speeds. The axis of oscillation during the test should be taken through the center of gravity location of the prototype, since in free flight the center of gravity travels nearly in a straight line for a stable missile (neglecting gravity). The center of gravity of the wind tunnel model should also be approximately on this axis of rotation, so that model oscillations do not cause large sting oscillations. In order that the aerodynamic damping be not masked by friction, the usual method is to use a crossed flexure pivot instead of ball bearings in the pivot, although ball bearings are used successfully. Some method is needed of obtaining an electrical signal proportional to amplitude of angular displacement; resistance strain gages work well on the flexures of a flexure pivot. This allows the amplitude-time history of the angular deflection of the model to be recorded. The time history of decaying oscillations is recorded both with wind on, and with wind off (generally with the tunnel evacuated). From these observations, it is hoped that the aerodynamic damping which would occur in free flight can be estimated.

GENERAL DISCUSSION

There is no good theory for calculating aerodynamic damping of oscillations. In general, consider the force on the tail fin of a missile or horizontal stabilizer tail surface of a plane instantaneously at zero pitch angle but with pitch angle rapidly increasing; this tail surface then has a forward velocity equal to that of the plane, and a downward velocity equal to the distance back from the c.g. times the angular velocity in pitch. This tail surface effectively has a positive angle of attack to the air, and there should then be a proportional lift (for small angles of attack). This lift on the tail surface furnishes a moment opposing the direction of angular velocity in pitch. From this crude reasoning, there is reason to expect an aerodynamic moment, proportional to the time rate of change of pitch angle and opposite in sign, thus resulting in a dissipation of energy.

If the missile is statically stable, there is an additional aerodynamic moment tending to restore it to the zero angle of pitch position, which for small angles may be directly proportional to the angle of attack.

The equation of motion of a body oscillating about an axis with angle of pitch θ, moment of inertia I, restoring moment $-k\theta$ and with damping moment $-\mu\dot{\theta}$ proportional to the angular velocity, is

(1) $$I\ddot{\theta} + \mu\dot{\theta} + k\theta = 0$$

which has the solution

(2) $$\theta = e^{-\lambda t} (A \sin \omega t + B \cos \omega t)$$

where

(3) $$\lambda = \frac{\mu}{2I} , \quad \omega = \sqrt{\frac{k}{I} - \frac{1}{4}\frac{\mu^2}{I^2}}$$

provided that $\mu^2 < 4kI$.

This solution (2) is the well-known exponentially decaying oscillation; the period or time for one oscillation is

(4) $$P = \frac{2\pi}{\omega} \doteq 2\pi \sqrt{\frac{I}{k}}$$

since generally ω is approximately equal to $\sqrt{k/I}$, and the frequency of oscillation is therefore approximately

(5) $$f = \frac{1}{2\pi}\sqrt{k/I} = \omega/2\pi .$$

If $\theta_0, \theta_1, \theta_2, \theta_3, \ldots$ are the amplitudes at successive maxima of θ at the corresponding times $t_1, t_2, t_3, \ldots$, then from (2)

(6) $$\frac{\theta_m}{\theta_n} = e^{-\lambda(t_m - t_n)} = e^{-\lambda(m - n)P}$$

so that from (3)

(7) $$\mu = 2I\lambda = 2I \frac{\ln(\theta_n/\theta_m)}{t_m - t_n} .$$

Since $\ln 2 = 0.69315$, we can also write

$$\mu = \frac{1.38630\, I}{\Delta t_2} \tag{8}$$

where Δt_2 is the time required for the amplitude to decrease to half its original value. Also from (6), the ratio of successive maxima is

$$\frac{\theta_{n+1}}{\theta_n} = e^{-\lambda P} \tag{9}$$

so that the ratio of successive maxima is a constant.

The total energy in the oscillating system at any time is the sum of the kinetic energy plus that stored in the spring,

$$E = \frac{1}{2} I\dot{\theta}^2 + \frac{1}{2} k\theta^2 \; ;$$

and at the times of maximum amplitude when $\dot{\theta} = 0$, this becomes

$$E_n = \frac{1}{2} k\theta_o^{\,2}\, e^{-2\lambda nP} \tag{10}$$

at the end of the n^{th} period from the time of the occurrence of the amplitude maximum θ_o .

DISCUSSION ON THE ACTUAL LAWS OF MOTION

It is sometimes customary to assume that the damping, both the aerodynamic and tare parts, is a moment proportional to $\dot{\theta}$. Thus BRL Report No. 1078 by H. E. Maloy (ref. 1) states that the motion of the body may be expressed by the linear differential equation

$$I\ddot{\theta} + \mu_2\dot{\theta} + k_2\theta = 0 \quad . \tag{11}$$

Here

$$\mu_2 = \mu_o + \mu_1 \quad \text{and} \quad k_2 = k_o + k_1$$

where μ_o and k_o are aerodynamic parameters, μ_1 and k_1 are mechanical parameters. It is implied that these quantities μ_o, μ_1, k_o, k_1 are constant, so that the equation (11) can be integrated to give the simple decaying sinusoidal solution. It is further assumed that with the wind off, μ_o and k_o become zero, μ_1 and k_1 are unchanged, so that the equation of motion becomes

$$I\ddot{\theta} + \mu_1\dot{\theta} + k_1\theta = 0 \tag{12}$$

The hope is that by recording the curve of decaying oscillations, measuring frequency and rate of decay for wind on and wind off, one can find μ_2, k_2 and μ_1, k_1; then by difference μ_o and k_o are found.

In ref. 1 Mr. Maloy noted that when he varied the moment of inertia in the wind-off tests, the resulting tare damping varied with frequency, and also to some extent with amplitude; he varied the moment of inertia of the model to get μ_1 for a range of frequencies, and used the value of μ_1 corresponding to the frequency of the wind-on run to subtract from μ_2 in order to get μ_o, the aerodynamic damping.

Of course these results mean that (12) and (11) are not the correct equations of motion; the correct equations of motion are <u>not</u> linear differential equations with constant coefficients, which yield the results in Mr. Maloy's tests. An equation of the form (12) yields a solution of general sinusoidal character with successive maxima of α decreasing exponentially; but the converse is not necessarily true. The fact that the successive maxima of α decrease exponentially does not at all imply that the differential equation of motion is like equation (12).

When the moment of inertia of the model was changed during the wind-off experiments and equation (7) or (8) used to find μ, the product μf turned out to be a constant, so that μ varied inversely as the frequency instead of being a constant. Thus for the curve for μ versus f in fig. 9, ref. 1, the product μf is 10 within the experimental accuracy. In a classified report (ref. 2) by M. A. Sylvester at the BRL wind tunnel, over a wider range of frequency, this constancy of the product μf was even more striking.

The energy of the oscillating model at any instant is composed of two parts, $1/2\ k\theta^2$ (potential energy stored in the spring) and $1/2\ I\dot{\theta}^2$ (kinetic energy). During an oscillation there is a transfer of energy from all potential energy at the extremes of amplitude to all kinetic energy at the times when $\theta = 0$. If the successive maxima θ_1, θ_2, θ_3, ... occur at times t_1, t_2, t_3, ..., then the loss of energy during the n^{th} cycle is

$$(13) \qquad \Delta E_n = E_n - E_{n+1} = \frac{1}{2}k(\theta_n^{\ 2} - \theta_{n+1}^{\ 2})$$

where k is the spring constant.

From the fact that the amplitude of oscillations slowly decays exponentially and that the product of μf is a constant as I only is changed, we have

$$(14) \qquad \begin{aligned} c &= \mu f \\ &= 2I\,\frac{\ell n\ \theta_n/\theta_{n+1}}{\Delta t_c}\,f \\ &= 2I\ f^2 \ell n\left(1 + \frac{\theta_n - \theta_{n+1}}{\theta_{n+1}}\right) \qquad \text{since } \Delta t_c = 1/f \\ &\doteq 2I\left(\frac{I}{4\pi^2}\,\frac{k}{I}\right)\frac{\theta_n - \theta_{n+1}}{\theta_{n+1}} \qquad \text{since } \Delta\theta_n \ll \theta_n \\ &= \frac{1}{\pi^2}\,\frac{\frac{1}{2}k(\theta_n^{\ 2} - \theta_{n+1}^{\ 2})}{\theta_{n+1}(\theta_n - \theta_{n+1})} \end{aligned}$$

Then from (13) this becomes

$$c = -\frac{1}{\pi^2}\frac{\Delta E_n}{\theta_{n+1}(\theta_n+\theta_{n+1})}$$

from which

$$\Delta E_n = -\pi^2 c\, \theta_n(\theta_n+\theta_{n+1})$$

$$= -\frac{2\pi^2 c}{k}\left(\frac{1}{2}k\theta_{n+1}^{\ 2}\right)\frac{(\theta_n+\theta_{n+1})}{\theta_{n+1}}$$

and therefore

(15) $$\Delta E_n \doteq 4\pi^2\frac{c}{k}E_{n+1} \quad .$$

The form of this equation, $\Delta E/E$ = a constant, is typical of functions decreasing exponentially with time if the period is independent of amplitude.

Thus the experimental result seems to lead to the result that the energy loss per cycle is independent of the frequency or of the moment of inertia. This energy loss may be due to hysteresis in the springs; such a possibility seems more probable than the assumption that the tare damping moment is proportional to $\dot{\theta}$.

If one assumes for the wind-off oscillations the following equation (for a spring with hysteresis proportional to the amplitude)

(16) $$I\ddot{\theta} = -k\theta+(\operatorname{sgn}\dot{\theta})\epsilon\theta_n \qquad \text{where } \operatorname{sgn}\dot{\theta} = \dot{\theta}/\left|\dot{\theta}\right|$$

for $t_n < t < t_{n+1}$, so that

$$I\ddot{\theta} + k\theta = \begin{cases} +\epsilon\theta_n & \text{for } t_n < t < t_n + \frac{1}{2}t_c \\ -\epsilon\theta_n & \text{for } t_n + \frac{1}{2}t_c < t < t_{n+1} \end{cases}$$

then the energy loss per cycle is

$$\Delta E_n \doteq \int -(\mathrm{sgn}\ \dot{\theta})\epsilon\theta_n d\theta = -\int_{+\theta_n}^{-\theta_n} \epsilon\theta_n d\theta + \int_{-\theta_n}^{\theta_n} \epsilon\theta_n d\theta$$

$$\doteq 4\epsilon\theta_n^2 \doteq \frac{8\epsilon}{k} E_n \tag{17}$$

Thus the loss per cycle is proportional to E_n, the energy present at the beginning of the cycle. Thus it appears that the fact that the amplitude decreases exponentially does not mean that the damping is of viscous nature (with a moment proportional to $\dot{\theta}$). It is of course now evident that the aerodynamic damping moment at any instant may be widely different in actual form from the assumption that it is directly proportional to $\dot{\theta}$. Since it would be exceedingly difficult to measure instantaneous values of damping moment at different points in the cycle of oscillation, it will doubtless continue to be the custom to observe only integrated effects over a cycle or several cycles, or to observe the time for oscillations to decay to half-amplitude, and then to define a μ as if the damping moment were $\mu\dot{\theta}$.

GENERAL COMMENTS ON SOURCES OF ERROR

If crossed flexures are used as a pivot and spring combined, it is very much better that this crossed flexure pivot be machined from one piece of metal. The experience of the Ballistic Research Laboratories at Aberdeen, Maryland has been that it is practically impossible to assemble a flexure pivot from simple flexures held together with screws, dowels and cement in such a manner that the tare damping would be reproducible over several runs; there always seems to be some slipping at joints, causing non-repeatable energy losses.

The tare damping seems to partially depend on balance-model, balance-sting, and sting-angle of attack mechanism attachments, so that it seems to be necessary to mount the model in the tunnel to measure the tare damping, or at least to mount model and sting on the actual angle of attack mechanism of the tunnel.

If mounted in the tunnel, the test section should be evacuated to a very low pressure or results extrapolated to zero pressure to remove the influence of surrounding still air on tare damping. However, this effect is usually small on most missile models.

The spring constant of a crossed-flexure pivot may depend on the amount of load on the flexure and on its direction. In tests of high-drag shapes such as some re-entry vehicles, this may be particularly important. Thus the spring constant k must be measured under both loading conditions, wind-on and wind-off.

If ball-bearing pivots are used, the energy loss per cycle of oscillation will certainly be a function of bearing load; this may be true of losses due to hysteresis of the metal in crossed-flexure pivots, if bearing loads produce appreciable strains in any port. Thus the tare damping should be checked to see if bearing loads have any effect on energy loss per cycle; if necessary, the tare damping must be measured when the bearing loads are the same as in the wind-on condition.

MODELS WITH VERY HIGH DAMPING

In some models, aerodynamic damping may be so great that there are no oscillations after the model is released, or perhaps all visible motion ceases after 2 to 5 oscillations. In this case, one can probably best make use of an analog computer[4] set to solve equation (1). The moment of inertia of the model must be measured and added to that of the movable part of the flexure pivot to get I, and guesses at μ and k inserted in the analog computer until the solution is a good fit with the oscillogram of θ vs. t obtained in the wind tunnel test. It may be assumed that the equation of motion is like equation (11) except that the tare damping is negligible:

(18) $$I\ddot{\theta} + \mu\dot{\theta} + k_2\theta = 0$$

where $k_2 = k_o + k_1$ with k_1 the mechanical spring constant and where $k_o\theta$ is the aerodynamic static stability moment.

If constants μ and k_2 can be found so that the analog computer solution of equation (18) agrees with the experimentally recorded decay of oscillation amplitude in the tunnel, then μ and $k_o = k_2 - k_1$ are the desired damping and static stability constants. If a good fit between the integration of equation (18) with the experimentally obtained damping curve cannot be obtained, then either the aerodynamic damping is not really proportional to $\mu\dot{\theta}$, or the aerodynamic static stability moment is not proportional to $k_o\theta$, so that the motion does not obey a simple equation with constant coefficients like (18).

RECOMMENDED PROCEDURE FOR NORMAL MISSILE TESTS

1. Equipment Design

In the normal missile test, the damping will be low enough so that the model will execute a large number of oscillations before the amplitude gets small. An essential requirement for a successful test is that the tunnel flow be sufficiently quiet so that flow irregularities will not keep the flexibly mounted model in motion. Since the measured damping may depend on the frequency, it is best that the wave length of oscillation in model lengths in the wind tunnel test be close to that in free flight--wave length being the distance in model lengths of relative motion of air to model for one model oscillation. If the static stability of the model (k_o) has been measured in previous wind tunnel tests and the moment of inertia of the prototype about its c.g. is known, then the expected frequency and therefore wave length for the free flight of the prototype is known. Then in the wind tunnel tests for damping, the flexure pivot (or other pivot) spring constant k_1 and the moment of inertia of the model should be selected to give the same wave length of oscillations: the period or time of one oscillation of the wind tunnel model in the wind-on tests will be approximately from equation (4)

$$P = 2\pi \sqrt{I/(k_0 + k_1)} \quad . \tag{19}$$

As stated earlier, it is recommended that crossed-flexure pivots machined of one piece of metal be used. In addition, great care needs to be taken in model-flexure, flexure-sting and sting-base attachments, so that small slippages there will not cause unpredictable energy losses. Care should be taken in the strain-gage mounting and disposition of the strain-gage lead wires, for the same reason. The model c.g. should be on or near the flexure axis, in order to reduce sting motion. If the model contains any adjustable weights, they must also be secured in such a way that slight slippage cannot occur to cause unpredictable or unrepeatable energy losses.

2. Tare Damping Experiments

Before running the wind-on tests, it is best to make a number of tare-damping recordings, to be sure that the tare losses are repeatable and consistent. These should include removing and replacing the model from the flexure, the flexure from the sting and the sting from the tunnel between runs, until the tare damping runs are shown to be consistent. The spring constant k_1 of the flexure pivot should be measured; it should be measured again with the sting pointed vertically upward and with weight added to represent the wind-on axial aerodynamic load, to check if k_1 varies with axial load, and k_1 should also be measured with the same normal load as the weight of the model will apply in the wind-on tests. In most cases, k_1 will be found not to vary appreciably with loads.

Since the frequency will be different in the wind-on tests and wind-off tests made with the same I, it is necessary to establish how the energy losses causing tare damping vary with frequency, so that the tare portion of energy loss can be known in the wind-on experiment. It is to be hoped that further experience will show the same results apparently obtained in references 1 and 2, namely:

> <u>The tare energy loss per cycle is a constant fraction of the energy stored in the spring at the beginning of the cycle, independent of the I of the model and hence its frequency</u>. This is equivalent to the relation μf = constant where μ is the viscous type damping constant giving the observed rate of amplitude decay.

If sufficient experience permits this rule to be safely accepted, it will no longer be necessary to carry out the tare damping tests at a series of different values of I and hence at different frequencies, as was done in references 1 and 2. The NASA method[3] of handling tare damping seems to be to compute the energy loss per cycle for the tare damping, and to reduce the amplitude loss per cycle in the wind-on runs by the corresponding amount to compute the corrected aerodynamic damping. This method would be correct if the energy loss per cycle in the wind-off tests is independent of frequency, as it seems to be in our tests. Of course, the method used in the past at BRL, to find the tare damping at the same frequency as that in the wind-on test, is also correct; although the "tare damping coefficient μ_1" corresponds to a fictitious damping moment $\mu_1\dot{\theta}$ or equivalent viscous damping which would give the same rate of decay of oscillations.

<u>Data Reduction</u> (<u>tare damping</u>): the observed successive maxima of amplitude should be plotted on semi-log paper as a function of time, in a region around the amplitude in which the aerodynamic damping is to be later determined, or $\ell n\ \theta_1$, $\ell n\ \theta_2$, ... , $\ell n\ \theta_n$, ... , should be plotted vs. time. The result should be nearly a straight line. The best tangent line should be drawn to this curve, at the selected amplitude. The time elapsed Δt_n for n cycles in this portion of the curve will give the frequency

(20) $$f_1 = n/\Delta t_n \quad \text{cycles per second}$$

Equation (5) can be solved for I to give

(21) $$I = k_1/(2\pi f_1)^2$$

giving the moment of inertia of the model, and the (fictitious) tare damping found from equation (7), as

(22) $$\mu_1 = \frac{2\ I}{t_m - t_n}\ \ell n \frac{\theta_n}{\theta_m}$$

If masses can be changed in the model to change the moment of inertia, then runs at different frequency should be reduced to check if the fictitious damping constant obeys the relation

(23) $$\mu_1 f = \text{constant as I varies.}$$

3. Wind-on Damping Experiments

The decaying oscillation curves should be run, and recorded via oscillograph. Again, successive maxima of amplitude should be measured, a plot made of $\ell n\theta_n$ vs. time, and the best straight line drawn in the neighborhood of the desired amplitude (or the best straight line through perhaps ten successive values of ($\ell n\theta_i$, t_i) found by computers from least squares methods). Then the frequency is found from

(24) $$f_2 = (m-n)/(t_m - t_n)$$

and the combined spring constant (see equation (5)) is computed from

(25) $$k_2 = I(2\pi f_2)^2 \quad .$$

By difference, the aerodynamic spring constant is found from

(26) $$k_o = k_2 - k_1$$

The combined damping constant μ_2 is found from (22) using wind-on values of θ_1 and t_1 . Finally, assuming that the tare damping varies inversely with frequency so that

$$\mu_{1t} f_t = \mu_{1c} \times f_w \quad ,$$

where the subscript t denotes quantities measured during a tare run, f_w denotes the frequency during the wind-on run, and μ_{1c} is the corrected equivalent viscous damping. Solving, this gives

$$\mu_{1c} = \mu_{1t} \frac{f_t}{f_w}$$

and the final aerodynamic damping coefficient is computed from

$$\mu_o = \mu_2 - \mu_{1c} = \mu_2 - \mu_{1t} f_t / f_w \tag{27}$$

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CREDITS

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Aerodynamic
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